Applied Mathematics and Mechanics
(English Edition, Vol 26, No 1, Jan 2005)

Published by Shanghai University,
Shanghai, China

Article ID: 0253-4827(2005)01-0017-10

SCATTERING OF HARMONIC ANTI-PLANE SHEAR WAVES BY AN INTERFACE CRACK IN MAGNETO-ELECTRO-ELASTIC COMPOSITES *

ZHOU Zhen-gong (周振功), WANG Biao (王 彪)

(Center for Composite Materials, Harbin Institute of Technology,
Harbin 150001, P.R.China)

(Contributed by WANG Biao)

Abstract: *The dynamic behavior of an interface crack in magneto-electro-elastic composites under harmonic elastic anti-plane shear waves is investigated for the permeable electric boundary conditions. By using the Fourier transform, the problem can be solved with a pair of dual integral equations in which the unknown variable was the jump of the displacements across the crack surfaces. To solve the dual integral equations, the jump of the displacements across the crack surface was expanded in a series of Jacobi polynomials. Numerical examples were provided to show the effect of the length of the crack, the wave velocity and the circular frequency of the incident wave on the stress, the electric displacement and the magnetic flux intensity factors of the crack. From the results, it can be obtained that the singular stresses in piezoelectric/piezomagnetic materials carry the same forms as those in a general elastic material for anti-plane shear problem.*

Key words: interface crack; elastic wave; magneto-electro-elastic composite; dual integral equation

Chinese Library Classification: O346.58 **Document code**: A

2000 Mathematics Subject Classification: 74J20; 74R10; 74F17

Introduction

Composite material consisting of a piezoelectric phase and a piezomagnetic phase has drawn significant interest in recent years, due to the rapid development in adaptive material systems. It shows a remarkably large magnetoelectric coefficient, the coupling coefficient between static electric and magnetic fields, which does not exist in either constituent. The magnetoelectric coupling is a new product property of the composite, since it is absent in each constituent. In some cases, the coupling effect of piezoelectric/piezomagnetic composites can be even obtained a hundred times larger than that in a single-phase magnetoelectric material. Consequently, they are

---

* **Received date**: 2003-10-30; **Revised date**: 2004-09-08

Foundation items: the National Natural Science Foundation of China (50232030, 10172030); the Natural Science Foundation of Heilongjiang Province of China (A0301)

Biography: ZHOU Zhen-gong, Professor, Doctor, E-mail: zhouzhg@hit.edu.cn

extensively used as electric packaging, sensors and actuators, e.g., magnetic field probes, acoustic/ultrasonic devices, hydrophones, and transducers with the responsibility of electro-magneto-mechanical energy conversion[1]. When subjected to mechanical, magnetic and electrical loads in service, these magneto-electro-elastic composites can fail prematurely due to some defects, e.g. cracks, holes, etc. arising during their manufacturing process. Therefore, it is of great importance to study the magneto-electro-elastic interaction and fracture behavior of magneto-electro-elastic composites[2,3].

The development of piezoelectric-piezomagnetic composites has its roots in the early work of Van Suchtelen[4] who proposed that the combination of piezoelectric-piezomagnetic phases may exhibit a new material property—the magnetoelectric coupling effect. Since then, the magnetoelectric coupling effect of $BaTiO_3$-$CoFe_2O_4$ composites has been measured by many researchers. Much of the theoretical work for the investigation of magnetoelectric coupling effect has only recently been studied[1~3,5~13]. To our knowledge, the magneto-electro-elastic dynamic behavior of magneto-electro-elastic composites with an interface crack subjected to harmonic anti-plane shear waves has not been studied.

In this paper, the behavior of an interface crack in magneto-electro-elastic composites subjected to under harmonic elastic anti-plane shear waves is investigated for the permeable electric boundary by use of a somewhat different method, named as the Schmidt method[14,15]. The Fourier transform is applied and a mixed boundary value problem is reduced to a pair of dual integral equations. To solve the dual integral equations, the jump of the displacements across the crack surfaces is expanded in a series of Jacobi polynomials. This process is quite different from those adopted in Refs.[2,3] as mentioned above. Numerical solutions are obtained for the stress intensity factors.

1 Formulation of the Problem

It is assumed that there is an interface crack of length $2l$ between two dissimilar magneto-electro-elastic composite half planes as shown in Fig.1. In this paper, the harmonic elastic anti-plane shear stress wave is vertically incident. Let ω be the circular frequency of the incident wave. $-\tau_0$ is a magnitude of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $e^{-i\omega t}$ will be suppressed but understood. The piezoelectric/piezomagnetic boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement, the in-plane electric and the in-plane magnetic fields. So the boundary conditions of the present problem are (In this paper, we just consider the perturbation fields).

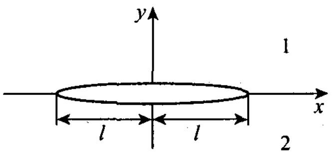

Fig.1 An interface crack between two dissimilar magneto-electro-elastic composite half planes

$$\begin{cases} \tau_{yz}^{(1)}(x,0^+) = \tau_{yz}^{(2)}(x,0^-) = -\tau_0 & (|x| \leqslant l), \\ w^{(1)}(x,0^+) = w^{(2)}(x,0^-) & (|x| > l); \end{cases} \tag{1}$$

$$\begin{cases} \phi^{(1)}(x,0^+) = \phi^{(2)}(x,0^-) \\ D_y^{(1)}(x,0^+) = D_y^{(2)}(x,0^-) \end{cases} \quad (|x| \leqslant \infty); \tag{2}$$

$$\begin{cases} \psi^{(1)}(x,0^+) = \psi^{(2)}(x,0^-) \\ B_y^{(1)}(x,0^+) = B_y^{(2)}(x,0^-) \end{cases} \quad (|x| \leqslant \infty); \tag{3}$$

$$w^{(1)}(x,y) = w^{(2)}(x,y) = 0 \quad \text{for} \quad (x^2+y^2)^{1/2} \to \infty, \tag{4}$$

where $\tau_{zk}^{(i)}, D_k^{(i)}$ and $B_k^{(i)}(k = x, y, i = 1, 2)$ are the anti-plane shear stress, in-plane electric displacement and in-plane magnetic flux, respectively. $w^{(i)}, \phi^{(i)}$ and $\psi^{(i)}$ are the mechanical displacement, the electric potential and the magnetic potential, respectively. Note that all quantities with superscript $i(i = 1,2)$ refer to the upper half plane 1 and the lower half plane 2 as in Fig.1, respectively. In this paper, we only consider that τ_0 is positive.

The constitutive equations can be written as

$$\tau_{zk}^{(i)} = c_{44}^{(i)} w_{,k}^{(i)} + e_{15}^{(i)} \phi_{,k}^{(i)} + q_{15}^{(i)} \psi_{,k}^{(i)} \quad (k = x, y; i = 1,2), \tag{5}$$

$$D_k^{(i)} = e_{15}^{(i)} w_{,k}^{(i)} - \varepsilon_{11}^{(i)} \phi_{,k}^{(i)} - d_{11}^{(i)} \psi_{,k}^{(i)} \quad (k = x, y; i = 1,2), \tag{6}$$

$$B_k^{(i)} = q_{15}^{(i)} w_{,k}^{(i)} - d_{11}^{(i)} \phi_{,k}^{(i)} - \mu_{11}^{(i)} \psi_{,k}^{(i)} \quad (k = x, y; i = 1,2), \tag{7}$$

where $c_{44}^{(i)}$ is shear modulus, $e_{15}^{(i)}$ is piezoelectric coefficient, $\varepsilon_{11}^{(i)}$ is dielectric parameter, $q_{15}^{(i)}$ is piezomagnetic coefficient, $d_{11}^{(i)}$ is electromagnetic coefficient, $\mu_{11}^{(i)}$ is magnetic permeability.

The anti-plane governing equations are

$$c_{44}^{(i)} \nabla^2 w^{(i)} + e_{15}^{(i)} \nabla^2 \phi^{(i)} + q_{15}^{(i)} \nabla^2 \psi^{(i)} = \rho^{(i)} \frac{\partial^2 w^{(i)}}{\partial t^2} \quad (i = 1,2), \tag{8}$$

$$e_{15}^{(i)} \nabla^2 w^{(i)} - \varepsilon_{11}^{(i)} \nabla^2 \phi^{(i)} - d_{11}^{(i)} \nabla^2 \psi^{(i)} = 0 \quad (i = 1,2), \tag{9}$$

$$q_{15}^{(i)} \nabla^2 w^{(i)} - d_{11}^{(i)} \nabla^2 \phi^{(i)} - \mu_{11}^{(i)} \nabla^2 \psi^{(i)} = 0 \quad (i = 1,2), \tag{10}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator. $\rho^{(i)}$ is the density of the piezoelectric/piezomagnetic materials. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0 \leqslant x < \infty$, $-\infty \leqslant y < \infty$ only. A Fourier transform is applied to Eqs.(8) ~ (10). It is assumed that the solutions are

$$\begin{cases} w^{(1)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_1(s)\mathrm{e}^{-\gamma_1 y}\cos(sx)\mathrm{d}s, \\ \phi^{(1)}(x,y) = \dfrac{\mu_{11}^{(1)} e_{15}^{(1)} - d_{11}^{(1)} q_{15}^{(1)}}{\varepsilon_{11}^{(1)} \mu_{11}^{(1)} - d_{11}^{(1)2}} w^{(1)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_1(s)\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s, \\ \psi^{(1)}(x,y) = \dfrac{q_{15}^{(1)} \varepsilon_{11}^{(1)} - d_{11}^{(1)} e_{15}^{(1)}}{\varepsilon_{11}^{(1)} \mu_{11}^{(1)} - d_{11}^{(1)2}} w^{(1)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_1(s)\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s \end{cases}$$

$$(y \geqslant 0); \tag{11}$$

$$\begin{cases} w^{(2)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_2(s)\mathrm{e}^{\gamma_2 y}\cos(sx)\mathrm{d}s, \\ \phi^{(2)}(x,y) = \dfrac{\mu_{11}^{(2)} e_{15}^{(2)} - d_{11}^{(2)} q_{15}^{(2)}}{\varepsilon_{11}^{(2)} \mu_{11}^{(2)} - d_{11}^{(2)2}} w^{(2)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_2(s)\mathrm{e}^{sy}\cos(sx)\mathrm{d}s, \\ \psi^{(2)}(x,y) = \dfrac{q_{15}^{(2)} \varepsilon_{11}^{(2)} - d_{11}^{(2)} e_{15}^{(2)}}{\varepsilon_{11}^{(2)} \mu_{11}^{(2)} - d_{11}^{(2)2}} w^{(2)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_2(s)\mathrm{e}^{sy}\cos(sx)\mathrm{d}s \end{cases}$$

$$(y \leqslant 0), \tag{12}$$

where $A_1(s), B_1(s), C_1(s), A_2(s), B_2(s)$ and $C_2(s)$ are unknown functions.

$$\gamma_1^2 = s^2 - \omega^2/c_1^2, \quad c_1^2 = \mu^{(1)}/\rho^{(1)}, \quad \mu^{(1)} = c_{44}^{(1)} + \frac{a_1 e_{15}^{(1)}}{a_0} + \frac{a_2 q_{15}^{(1)}}{a_0},$$

$$a_0 = \varepsilon_{11}^{(1)}\mu_{11}^{(1)} - d_{11}^{(1)2},\ a_1 = \mu_{11}^{(1)}e_{15}^{(1)} - d_{11}^{(1)}q_{15}^{(1)},\ a_2 = q_{15}^{(1)}\varepsilon_{11}^{(1)} - d_{11}^{(1)}e_{15}^{(1)},$$

$$\gamma_2^2 = s^2 - \omega^2/c_2^2,\ c_2^2 = \mu^{(2)}/\rho^{(2)},\ \mu^{(2)} = c_{44}^{(2)} + \frac{a_4 e_{15}^{(2)}}{a_3} + \frac{a_5 q_{15}^{(2)}}{a_3},$$

$$a_3 = \varepsilon_{11}^{(2)}\mu_{11}^{(2)} - d_{11}^{(2)^2},\ a_4 = \mu_{11}^{(2)}e_{15}^{(2)} - d_{11}^{(2)}q_{15}^{(2)},\ a_5 = q_{15}^{(2)}\varepsilon_{11}^{(2)} - d_{11}^{(2)}e_{15}^{(2)}.$$

So from Eqs. (5) ~ (7), we have

$$\tau_{yz}^{(1)}(x,y) = -\frac{2}{\pi}\int_0^\infty \Big\{\gamma_1\Big(c_{44}^{(1)} + \frac{a_1 e_{15}^{(1)}}{a_0} + \frac{a_2 q_{15}^{(1)}}{a_0}\Big)A_1(s)e^{-\gamma_1 y} + s[e_{15}^{(1)}B_1(s) + q_{15}^{(1)}C_1(s)]e^{-sy}\Big\}\cos(sx)ds, \tag{13}$$

$$D_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty s[\varepsilon_{11}^{(1)}B_1(s) + d_{11}^{(1)}C_1(s)]e^{-sy}\cos(sx)ds, \tag{14}$$

$$B_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty s[d_{11}^{(1)}B_1(s) + \mu_{11}^{(1)}C_1(s)]e^{-sy}\cos(sx)ds, \tag{15}$$

$$\tau_{yz}^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty \Big\{\gamma_2\Big(c_{44}^{(2)} + \frac{a_4 e_{15}^{(2)}}{a_3} + \frac{a_5 q_{15}^{(2)}}{a_3}\Big)A_2(s)e^{\gamma_2 y} + s[e_{15}^{(2)}B_2(s) + q_{15}^{(2)}C_2(s)]e^{sy}\Big\}\cos(sx)ds, \tag{16}$$

$$D_y^{(2)}(x,y) = -\frac{2}{\pi}\int_0^\infty s[\varepsilon_{11}^{(2)}B_2(s) + d_{11}^{(2)}C_2(s)]e^{sy}\cos(sx)ds, \tag{17}$$

$$B_y^{(2)}(x,y) = -\frac{2}{\pi}\int_0^\infty s[d_{11}^{(2)}B_2(s) + \mu_{11}^{(2)}C_2(s)]e^{sy}\cos(sx)ds. \tag{18}$$

To solve the problem, the jumps of the displacements, the electric and the magnetic potentials across the crack surfaces are defined as follows:

$$f(x) = w^{(1)}(x,0^+) - w^{(2)}(x,0^-). \tag{19}$$

Substituting Eqs. (11) ~ (12) into Eq. (19), and applying the Fourier transform and the boundary conditions (2) ~ (3), we can obtain

$$\bar{f}(s) = A_1(s) - A_2(s), \tag{20}$$

$$\frac{a_1}{a_0}A_1(s) - \frac{a_4}{a_3}A_2(s) + B_1(s) - B_2(s) = 0, \tag{21}$$

$$\frac{a_2}{a_0}A_1(s) - \frac{a_5}{a_3}A_2(s) + C_1(s) - C_2(s) = 0. \tag{22}$$

Substituting Eqs. (13) ~ (18) into Eqs. (1) ~ (3), we can obtain

$$-\Big(c_{44}^{(1)} + \frac{a_1 e_{15}^{(1)}}{a_0} + \frac{a_2 q_{15}^{(1)}}{a_0}\Big)\gamma_1 A_1(s) - se_{15}^{(1)}B_1(s) - sq_{15}^{(1)}C_1(s) - \Big(c_{44}^{(2)} + \frac{a_4 e_{15}^{(2)}}{a_3} + \frac{a_5 q_{15}^{(2)}}{a_3}\Big)\gamma_2 A_2(s) - se_{15}^{(2)}B_2(s) - sq_{15}^{(2)}C_2(s) = 0, \tag{23}$$

$$\varepsilon_{11}^{(1)}B_1(s) + d_{11}^{(1)}C_1(s) + \varepsilon_{11}^{(2)}B_2(s) + d_{11}^{(2)}C_2(s) = 0, \tag{24}$$

$$d_{11}^{(1)}B_1(s) + \mu_{11}^{(1)}C_1(s) + d_{11}^{(2)}B_2(s) + \mu_{11}^{(2)}C_2(s) = 0. \tag{25}$$

By solving six Eqs. (20) ~ (25) with six unknown functions $A_1(s), B_1(s), C_1(s), A_2(s), B_2(s), C_2(s)$ and applying the boundary condition (1) to the results, we can obtain:

$$\frac{2}{\pi}\int_0^\infty g_1(s)\bar{f}(s)\cos(sx)ds = -\tau_0 \qquad (0 \leqslant x \leqslant l), \tag{26}$$

$$\int_0^\infty \bar{f}(s)\cos(sx)\mathrm{d}s = 0 \qquad (x > l), \tag{27}$$

where $g_1(s)$ is a known function (see Appendix). $\lim_{s\to\infty} g_1(s)/s = \beta_1$. Where β_1 is a constant, which depends on the properties of the materials (see Appendix). When the properties of the upper and the lower half planes is the same, $\beta_1 = -c_{44}^{(1)}/2$. To determine the unknown functions $\bar{f}(s)$, the dual-integral equations (26) and (27) must be solved.

2 Solution of the Dual Integral Equations

To solve the dual integral equations (26) and (27), the jump of the displacements across the crack surfaces is represented by the following series:

$$f(x) = \sum_{n=1}^\infty b_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1 - \frac{x^2}{l^2}\right)^{1/2} \qquad (0 \leqslant x \leqslant l), \tag{28}$$

where b_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ are a Jacobi polynomial[16].

The Fourier transform of Eq.(28) is[17]

$$\bar{f}(s) = \sum_{n=1}^\infty b_n G_n \frac{1}{s}\mathrm{J}_{2n-1}(sl), \quad G_n = 2\sqrt{\pi}(-1)^{n-1}\frac{\Gamma(2n - 1/2)}{(2n-2)!}, \tag{29}$$

where $\Gamma(x)$ and $\mathrm{J}_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq.(29) into Eqs.(26) ~ (27), Eq.(27) has been automatically satisfied. After integration with respect to x in $[0,x]$, Eq.(26) reduces to

$$\sum_{n=1}^\infty b_n G_n \int_0^\infty \frac{g_1(s)}{s^2}\mathrm{J}_{2n-1}(sl)\sin(sx)\mathrm{d}s = -\frac{\pi\tau_0 x}{2}. \tag{30}$$

From the relationship[16]

$$\int_0^\infty \frac{1}{s}\mathrm{J}_n(sa)\sin(bs)\mathrm{d}s = \begin{cases} \dfrac{\sin[n\arcsin(b/a)]}{n} & (a > b), \\ \dfrac{a^n\sin(n\pi/2)}{n[b+\sqrt{b^2-a^2}]^n} & (b > a), \end{cases} \tag{31}$$

the semi-infinite integral in Eq.(30) can be modified as

$$\int_0^\infty \frac{1}{s}\left[\beta_1 + \left(\frac{g_1(s)}{s} - \beta_1\right)\right]\mathrm{J}_{2n-1}(sl)\sin(sx)\mathrm{d}s$$
$$= \frac{\beta_1}{2n-1}\sin\left[(2n-1)\arcsin\left(\frac{x}{l}\right)\right] + \int_0^\infty \frac{1}{s}\frac{g_1(s) - s\beta_1}{s}\mathrm{J}_{2n-1}(sl)\sin(sx)\mathrm{d}s.$$

Thus the semi-infinite integral in Eq.(30) can be evaluated directly. Equation (30) can now be solved for the coefficients b_n by the Schmidt method[14,15]. The method is omitted in the present work. It can be seen in Ref.[14].

3 Intensity Factors

The coefficients b_n are known, so that the entire perturbation dynamic stress field, the perturbation electric displacement and the magnetic flux can be obtained. However, in fracture mechanics, it is of importance to determine the perturbation stress $\tau_{yz}^{(1)}$, the perturbation electric displacement $D_y^{(1)}$ and the magnetic flux $B_y^{(1)}$ in the vicinity of the crack tips. In the case of the present study, $\tau_{yz}^{(1)}$, $D_y^{(1)}$ and $B_y^{(1)}$ along the crack line can be expressed respectively as

$$\tau_{yz}^{(1)}(x,0) = \frac{2}{\pi}\sum_{n=1}^\infty b_n G_n \int_0^\infty \frac{g_1(s)}{s}\mathrm{J}_{2n-1}(sl)\cos(xs)\mathrm{d}s, \tag{32}$$

$$D_y^{(1)}(x,0) = \frac{2}{\pi}\sum_{n=1}^{\infty} b_n G_n \int_0^{\infty} \frac{g_2(s)}{s} J_{2n-1}(sl)\cos(xs)\mathrm{d}s, \tag{33}$$

$$B_y^{(1)}(x,0) = \frac{2}{\pi}\sum_{n=1}^{\infty} b_n G_n \int_0^{\infty} \frac{g_3(s)}{s} J_{2n-1}(sl)\cos(xs)\mathrm{d}s, \tag{34}$$

where $g_2(s)$ and $g_3(s)$ are known functions (see Appendix). $\lim\limits_{s\to\infty} g_2(s)/s = \beta_2$. $\lim\limits_{s\to\infty} g_3(s)/s = \beta_3$. Where β_2 and β_3 are two constants which depend on the properties of the materials (see Appendix). When the properties of the upper and the lower half planes is the same, $\beta_2 = -e_{15}^{(1)}/2$ and $\beta_3 = -q_{15}^{(1)}/2$. By examination Eqs. (32) ~ (34), the singular parts of the stress field, the electric displacement and the magnetic flux can be obtained respectively from the relationship[16]

$$\int_0^{\infty} J_n(sa)\sin(bs)\mathrm{d}s = \begin{cases} \dfrac{\cos[n\arcsin(b/a)]}{\sqrt{a^2-b^2}} & (a > b), \\ -\dfrac{a^n \sin(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n} & (b > a). \end{cases} \tag{35}$$

The singular parts of the stress field, the electric displacement and the magnetic flux can be expressed respectively as follows $(x > l)$:

$$\tau = -\frac{2\beta_1}{\pi}\sum_{n=1}^{\infty} b_n G_n H_n(x), \tag{36}$$

$$D = -\frac{2\beta_2}{\pi}\sum_{n=1}^{\infty} b_n G_n H_n(x), \tag{37}$$

$$B = -\frac{2\beta_3}{\pi}\sum_{n=1}^{\infty} b_n G_n H_n(x), \tag{38}$$

where $H_n(x) = \dfrac{(-1)^{n-1} l^{2n-1}}{\sqrt{x^2-l^2}[x+\sqrt{x^2-l^2}]^{2n-1}}$.

We obtain the stress intensity factor K as

$$K = \lim_{x\to l^+}\sqrt{2(x-l)}\cdot\tau = -\frac{4\beta_1}{\sqrt{\pi l}}\sum_{n=1}^{\infty} b_n \frac{\Gamma(2n-1/2)}{(2n-2)!}. \tag{39}$$

We obtain the electric displacement intensity factor K^D as

$$K^D = \lim_{x\to l^+}\sqrt{2(x-l)}\cdot D = -\frac{4\beta_2}{\sqrt{\pi l}}\sum_{n=1}^{\infty} b_n \frac{\Gamma(2n-1/2)}{(2n-2)!} = \frac{\beta_2}{\beta_1}K. \tag{40}$$

We obtain the magnetic flux intensity factor K^B as

$$K^B = \lim_{x\to l^+}\sqrt{2(x-l)}\cdot B = -\frac{4\beta_3}{\sqrt{\pi l}}\sum_{n=1}^{\infty} b_n \frac{\Gamma(2n-1/2)}{(2n-2)!} = \frac{\beta_3}{\beta_1}K. \tag{41}$$

4 Conclusions

As discussed in Refs. [12 ~ 15], it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series in Eq. (30) are retained. The behavior of the sum of the series keeps steady with the increasing number of terms in Eq. (30). The properties[3,9,10] of materials are assumed to be $c_{44}^{(1)} = 44.0(\mathrm{GPa})$, $e_{15}^{(1)} = 5.8(\mathrm{C/m^2})$, $\varepsilon_{11}^{(1)} = 5.64\times10^{-9}(\mathrm{C^2/Nm^2})$, $q_{15}^{(1)} = 275.0(\mathrm{N/Am})$, $d_{11}^{(1)} = 0.005\times10^{-1}(\mathrm{Ns/VC})$, $\mu_{11}^{(1)} = -297.0\times10^{-6}(\mathrm{Ns^2/C^2})$, $c_{44}^{(2)} = 54.0(\mathrm{GPa})$, $e_{15}^{(2)} = 7.8(\mathrm{C/m^2})$, $\varepsilon_{11}^{(2)} = 3.64\times$

$10^{-9}(C^2/Nm^2)$, $q_{15}^{(2)} = 175.0(N/Am)$, $d_{11}^{(2)} = 0.008 \times 10^{-9}(Ns/VC)$, $\mu_{11}^{(2)} = -197.0 \times 10^{-6}(Ns^2/C^2)$. At $-l \leqslant x \leqslant l, y = 0$, it can be obtained that $\tau_{yz}^{(1)}/\tau_0$ is very close to negative unity. Hence, the solution of present paper can also be proved to satisfy the boundary conditions (1). The numerical results of the present paper are shown in Figs.2 ~ 4.

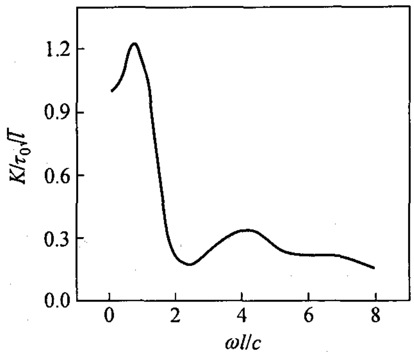

Fig.2 The stress intensity factor versus $\omega l/c$ for the interface crack

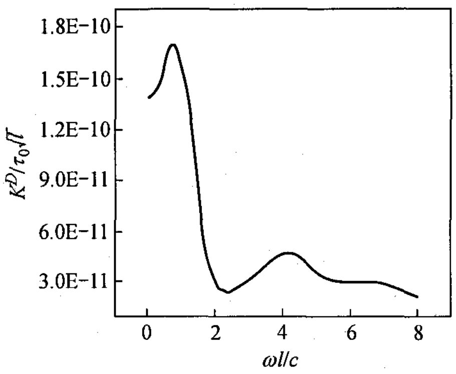

Fig. 3 The electric displacement intensity factor versus $\omega l/c$ for the interface crack

From the results, the following observations are very significant:

(i) The dynamic stress intensity factor depends on the material properties for the anti-plane shear interface crack fracture problem in magneto-electro-elastic composites. This is the same as the anti-plane shear fracture problem in the general elastic materials. The electro-magneto-elastic coupling effects can be obtained as shown in Eqs.(40) and (41). The electric displacement and the magnetic flux intensity factors depend on the length of the crack, the wave velocity, the circular frequency of the incident waves and the properties of the magneto-electro-elastic composite materials. It can be shown in Eqs.(40) and (41).

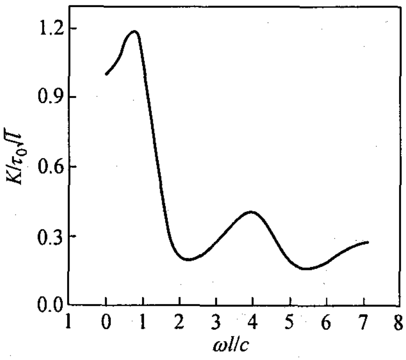

Fig.4 The stress intensity factor versus $\omega l/c$ for a crack in the homogeneous materials

(ii) The dynamic stress intensity factor tends to increase with the increase in the circular frequency of the incident waves, until reaching a maximum at $\omega l/c \approx 0.8$, then it decreases oscillating in magnitude as shown in Fig.2. For the electric displacement and the magnetic flux intensity factors, they have the same changing tendency with the frequency of the incident waves as the stress intensity factors as shown in Eqs. (40) and (41). The results of the electric displacement and the magnetic flux intensity factors can be directly obtained from the results of the stress intensity factors through Eqs.(40) and (41). Here, it is omitted.

(iii) The solution of this problem can be returned to the static solution for $\omega l/c = 0$. From the results, it can be shown that the stress intensity factor is equal to a unit when $\omega l/c = 0$ as shown in Fig.2. This is consistent with the fracture problem in the general elastic materials for the anti-plane shear fracture problem.

(ⅳ) When the properties of the upper and the lower half planes are the same, the numerical solution can be also obtained as shown in Fig.4. The stress intensity factor is equal to a unit when $\omega l/c = 0$ as shown in Fig.4. However, the maximum value of the stress intensity factors of the interface crack is larger than one of the cracks in homogeneous materials as shown in Fig.2 and Fig.4.

References:

[1] Wu T L, Huang J H. Closed-form solutions for the magnetoelectric coupling coefficients in fibrous composites with piezoelectric and piezomagnetic phases[J]. *International Journal of Solids and Structures*, 2000, **37**(21): 2981 - 3009.

[2] Sih G C, Song Z F. Magnetic and electric poling effects associated with crack growth in $BaTiO_3$-$CoFe_2O_4$ composite[J]. *Theoretical and Applied Fracture Mechanics*, 2003, **39**(3): 209 - 227.

[3] Song Z F, Sih G C. Crack initiation behavior in magnetoelectroelastic composite under in-plane deformation[J]. *Theoretical and Applied Fracture Mechanics*, 2003, **39**(3): 189 - 207.

[4] Van Suchtelen J. Product properties: a new application of composite materials[J]. *Phillips Research Reports*, 1972, **27**(1): 28 - 37.

[5] Harshe G, Dougherty J P, Newnham R E. Theoretical modeling of 3-0/0-3 magnetoelectric composites[J]. *International Journal of Applied Electromagnetics in Materials*, 1993, **4**(1): 161 - 171.

[6] Avellaneda M, Harshe G. Magnetoelectric effect in piezoelectric/magnetostrictive multiplayer (2-2) composites[J]. *Journal of Intelligent Material Systems and Structures*, 1994, **5**(3): 501 - 513.

[7] Nan Cewen. Magnetoelectric effect in composites of piezoelectric and piezomagnetic phases[J]. *Physical Review B*, 1994, **50**(20): 6082 - 6088.

[8] Benveniste Y. Magnetoelectric effect in fibrous composites with piezoelectric and magnetostrictive phases[J]. *Physical Review B*, 1995, **51**(8): 16424 - 16427.

[9] Huang J H, Kuo W S. The analysis of piezoelectric/piezomagnetic composite materials containing ellipsoidal inclusions[J]. *Journal of Applied Physics*, 1997, **81**(3): 1378 - 1386.

[10] Li J Y. Magnetoelectroelastic multi-inclusion and inhomogeneity problems and their applications in composite materials[J]. *International Journal of Engineering Science*, 2000, **38**(18): 1993 - 2011.

[11] Zhou Zhengong, Shen Yapeng. Investigation of the scattering of harmonic shear waves by two collinear cracks using the non-local theory[J]. *Acta Mechanica*, 1999, **135**(3/4): 169 - 179.

[12] Zhou Zhengong, Li Haichen. Investigation of the scattering of anti-plane shear waves by two collinear cracks in a piezoelectric materials using a new method[J]. *Acta Mechanica*, 2001, **147**(1-4): 87 - 97.

[13] Zhou Zhengong, Wang Biao. Investigation of anti-plane shear behavior of two collinear impermeable cracks in the piezoelectric materials by using the non-local theory[J]. *International Journal of Solids and Structures*, 2002, **39**(7): 1731 - 1742.

[14] Morse P M, Feshbach H. *Methods of Theoretical Physics*[M]. Vol 1. McGraw-Hill, New York, 1958, 926.

[15] Yan W F. Axisymmetric slipless indentation of an infinite elastic cylinder[J]. *SIAM Journal on Applied Mathematics*, 1967, **15**(2): 219 - 227.

[16] Gradshteyn I S, Ryzhik I M. *Table of Integrals, Series and Products*[M]. Academic Press, New York, 1980, 1035 - 1037.

[17] Erdelyi A. *Tables of Integral Transforms*[M]. McGraw-Hill, New York, 1954, 34 - 89.

Appendix

$$X_1 = \begin{bmatrix} 1 & 0 & 0 \\ \dfrac{a_1}{a_0} & 1 & 0 \\ \dfrac{a_2}{a_0} & 0 & 1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -1 & 0 & 0 \\ -\dfrac{a_4}{a_3} & -1 & 0 \\ -\dfrac{a_5}{a_3} & 0 & -1 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} -\left(c_{44}^{(1)} + \dfrac{a_1 e_{15}^{(1)}}{a_0} + \dfrac{a_2 q_{15}^{(1)}}{a_0}\right)\gamma_1 & -s e_{15}^{(1)} & -s q_{15}^{(1)} \\ 0 & \varepsilon_{11}^{(1)} & d_{11}^{(1)} \\ 0 & d_{11}^{(1)} & \mu_{11}^{(1)} \end{bmatrix},$$

$$X_4 = \begin{bmatrix} -\left(c_{44}^{(2)} + \dfrac{a_4 e_{15}^{(2)}}{a_3} + \dfrac{a_5 q_{15}^{(2)}}{a_3}\right)\gamma_2 & -s e_{15}^{(2)} & -s q_{15}^{(2)} \\ 0 & \varepsilon_{11}^{(2)} & d_{11}^{(2)} \\ 0 & d_{11}^{(2)} & \mu_{11}^{(2)} \end{bmatrix}, \quad X_5 = X_1 - X_2 X_4^{-1} X_3,$$

$$X_6 = \begin{bmatrix} x_{11}(s) & x_{12}(s) & x_{13}(s) \\ x_{21}(s) & x_{22}(s) & x_{23}(s) \\ x_{31}(s) & x_{32}(s) & x_{33}(s) \end{bmatrix} = X_3 X_5^{-1}.$$

$$g_1(s) = x_{11}(s),\ g_2(s) = x_{21}(s),\ g_3(s) = x_{31}(s).$$

$$H_1 = -2d_{11}^{(1)} e_{15}^{(1)} c_{44}^{(2)} q_{15}^{(1)} + e_{15}^{(2)^2} q_{15}^{(1)^2} + c_{44}^{(2)} \varepsilon_{11}^{(1)} q_{15}^{(1)^2} - 2e_{15}^{(1)} e_{15}^{(2)} q_{15}^{(1)} q_{15}^{(2)} + e_{15}^{(1)^2} q_{15}^{(2)^2} - 2e_{15}^{(1)} c_{44}^{(2)} q_{15}^{(1)} d_{11}^{(2)},$$

$$H_2 = c_{44}^{(2)} q_{15}^{(1)^2} \varepsilon_{11}^{(2)} + e_{15}^{(1)^2} c_{44}^{(2)} (\mu_{11}^{(2)} + \mu_{11}^{(1)}),$$

$$H_3 = c_{44}^{(1)} [-d_{11}^{(1)^2} c_{44}^{(2)} - 2e_{15}^{(2)} q_{15}^{(2)} d_{11}^{(2)} - c_{44}^{(2)} d_{11}^{(2)^2} - 2d_{11}^{(1)} (e_{15}^{(2)} q_{15}^{(2)} + c_{44}^{(2)} d_{11}^{(2)}) + q_{15}^{(2)^2} \varepsilon_{11}^{(2)}],$$

$$H_4 = c_{44}^{(1)} \{ e_{15}^{(2)^2} \mu_{11}^{(2)} + c_{44}^{(2)} \varepsilon_{11}^{(2)} \mu_{11}^{(2)} + e_{15}^{(2)^2} \mu_{11}^{(1)} + c_{44}^{(2)} \varepsilon_{11}^{(2)} \mu_{11}^{(1)} + \varepsilon_{11}^{(1)} [q_{15}^{(2)^2} + c_{44}^{(2)} (\mu_{11}^{(2)} + \mu_{11}^{(1)})]\},$$

$$R_1 = -d_{11}^{(1)^2} c_{44}^{(2)} + \varepsilon_{11}^{(1)} q_{15}^{(1)^2} + 2\varepsilon_{11}^{(1)} q_{15}^{(1)} q_{15}^{(2)} + \varepsilon_{11}^{(1)} q_{15}^{(2)^2} - 2e_{15}^{(1)} q_{15}^{(1)} d_{11}^{(2)} - 2e_{15}^{(2)} q_{15}^{(1)} d_{11}^{(2)},$$

$$R_2 = -2e_{15}^{(1)} q_{15}^{(2)} d_{11}^{(2)} - 2e_{15}^{(2)} q_{15}^{(2)} d_{11}^{(2)} - c_{44}^{(2)} d_{11}^{(2)^2},$$

$$R_3 = -2d_{11}^{(1)} [e_{15}^{(1)} (q_{15}^{(1)} + q_{15}^{(2)}) + e_{15}^{(2)} (q_{15}^{(1)} + q_{15}^{(2)}) + c_{44}^{(2)} d_{11}^{(2)}],$$

$$R_4 = q_{15}^{(1)^2} \varepsilon_{11}^{(2)} + 2q_{15}^{(1)} q_{15}^{(2)} \varepsilon_{11}^{(2)} + q_{15}^{(2)^2} \varepsilon_{11}^{(2)} + e_{15}^{(1)^2} \mu_{11}^{(2)} + 2e_{15}^{(1)} e_{15}^{(2)} \mu_{11}^{(2)} + e_{15}^{(2)^2} \mu_{11}^{(2)} + c_{44}^{(2)} \varepsilon_{11}^{(1)} \mu_{11}^{(2)},$$

$$R_5 = c_{44}^{(2)} \varepsilon_{11}^{(2)} \mu_{11}^{(2)} + e_{15}^{(1)^2} \mu_{11}^{(1)} + 2e_{15}^{(1)} e_{15}^{(2)} \mu_{11}^{(1)} + e_{15}^{(2)^2} \mu_{11}^{(1)} + c_{44}^{(2)} \varepsilon_{11}^{(1)} \mu_{11}^{(1)} + c_{44}^{(2)} \varepsilon_{11}^{(2)} \mu_{11}^{(1)},$$

$$R_6 = -c_{44}^{(1)} [d_{11}^{(1)^2} + 2d_{11}^{(1)} d_{11}^{(2)} + d_{11}^{(2)^2} - (\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)})(\mu_{11}^{(1)} + \mu_{11}^{(2)})],$$

$$\beta_1 = (H_1 + H_2 + H_3 + H_4)/(R_1 + R_2 + R_3 + R_4 + R_5 + R_6);$$

$$S_1 = -e_{15}^{(2)} \varepsilon_{11}^{(1)} q_{15}^{(1)^2} - e_{15}^{(2)} \varepsilon_{11}^{(1)} q_{15}^{(1)} q_{15}^{(2)} + e_{15}^{(1)} e_{15}^{(2)} q_{15}^{(1)} d_{11}^{(2)} - c_{44}^{(2)} \varepsilon_{11}^{(1)} q_{15}^{(1)} d_{11}^{(2)} + e_{15}^{(1)^2} q_{15}^{(2)} d_{11}^{(2)},$$

$$S_2 = 2e_{15}^{(1)} e_{15}^{(2)} q_{15}^{(2)} d_{11}^{(2)} + e_{15}^{(1)} c_{44}^{(2)} d_{11}^{(2)^2} - e_{15}^{(1)} q_{15}^{(1)} q_{15}^{(2)} \varepsilon_{11}^{(2)} - e_{15}^{(1)} q_{15}^{(2)^2} \varepsilon_{11}^{(2)},$$

$$S_3 = d_{11}^{(1)}\{e_{15}^{(1)}[e_{15}^{(2)}(2q_{15}^{(1)} + q_{15}^{(2)}) + c_{44}^{(2)}d_{11}^{(2)}] + q_{15}^{(1)}(e_{15}^{(2)^2} + c_{44}^{(2)}\varepsilon_{11}^{(2)}\},$$

$$S_4 = -e_{15}^{(1)^2}e_{15}^{(2)}\mu_{11}^{(2)} - e_{15}^{(1)^2}e_{15}^{(2)^2}\mu_{11}^{(2)} - e_{15}^{(1)}c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(2)} - e_{15}^{(1)^2}e_{15}^{(2)}\mu_{11}^{(1)} - e_{15}^{(1)}e_{15}^{(2)^2}\mu_{11}^{(1)} - e_{15}^{(1)}c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(1)},$$

$$S_5 = c_{44}^{(1)}\{d_{11}^{(1)^2}e_{15}^{(2)} + d_{11}^{(1)}(e_{15}^{(2)}d_{11}^{(2)} - q_{15}^{(2)}\varepsilon_{11}^{(2)}) + \varepsilon_{11}^{(1)}[q_{15}^{(2)}d_{11}^{(2)} - e_{15}^{(2)}(\mu_{11}^{(2)} + \mu_{11}^{(1)})]\},$$

$$\beta_2 = -(S_1 + S_2 + S_3 + S_4 + S_5)/(R_1 + R_2 + R_3 + R_4 + R_5 + R_6);$$

$$Y_1 = \varepsilon_{11}^{(1)}q_{15}^{(1)^2}q_{15}^{(2)} + \varepsilon_{11}^{(1)}q_{15}^{(1)}q_{15}^{(2)^2} - e_{15}^{(2)}q_{15}^{(1)^2}d_{11}^{(2)} - e_{15}^{(1)}q_{15}^{(1)}q_{15}^{(2)}d_{11}^{(2)} - 2e_{15}^{(2)}q_{15}^{(1)}q_{15}^{(2)}d_{11}^{(2)} - c_{44}^{(2)}q_{15}^{(1)}d_{11}^{(1)^2},$$

$$Y_2 = q_{15}^{(1)^2}q_{15}^{(2)}\varepsilon_{11}^{(2)} + q_{15}^{(1)}q_{15}^{(2)^2}\varepsilon_{11}^{(2)} + e_{15}^{(1)}e_{15}^{(2)}q_{15}^{(1)}\mu_{11}^{(2)} + e_{15}^{(2)^2}q_{15}^{(1)}\mu_{11}^{(2)} + c_{44}^{(2)}\varepsilon_{11}^{(1)}q_{15}^{(1)}\mu_{11}^{(2)} + c_{44}^{(2)}q_{15}^{(1)}\varepsilon_{11}^{(2)}\mu_{11}^{(2)},$$

$$Y_3 = -d_{11}^{(1)}(2e_{15}^{(1)}q_{15}^{(1)}q_{15}^{(2)} + e_{15}^{(2)}q_{15}^{(1)}q_{15}^{(2)} + e_{15}^{(1)}q_{15}^{(2)^2} + c_{44}^{(2)}q_{15}^{(1)}d_{11}^{(2)} + c_{44}^{(2)}e_{15}^{(1)}\mu_{11}^{(2)}),$$

$$Y_4 = e_{15}^{(1)^2}q_{15}^{(2)}\mu_{11}^{(1)} + e_{15}^{(1)}e_{15}^{(2)}q_{15}^{(2)}\mu_{11}^{(1)} + e_{15}^{(1)}c_{44}^{(2)}d_{11}^{(2)}\mu_{11}^{(1)},$$

$$Y_5 = c_{44}^{(1)}(-d_{11}^{(1)^2}q_{15}^{(2)} - d_{11}^{(1)}d_{11}^{(2)}q_{15}^{(2)} + d_{11}^{(1)}e_{15}^{(2)}\mu_{11}^{(2)} + \varepsilon_{11}^{(1)}q_{15}^{(2)}\mu_{11}^{(1)} - e_{15}^{(1)}d_{11}^{(2)}\mu_{11}^{(1)} + q_{15}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(1)}),$$

$$\beta_3 = (Y_1 + Y_2 + Y_3 + Y_4 + Y_5)/(R_1 + R_2 + R_3 + R_4 + R_5 + R_6).$$